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Tachyon condensates, carrollian contraction of Lorentz group, and fundamental strings

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Abstract: We study the rolling tachyon condensate in the presence of a gauge field. The generic vacuum admits both a rolling tachyon, $\dot{T}$, and a uniform electric field, $\vec{E}$, which together affect the effective metric governing the fluctuations of open string modes. If one suppresses the gauge field altogether, the light-cone collapses completely. This is the carrollian limit, with vanishing speed of light and no possible propagation of signals. In the presence of a gauge field, however, the lightcone is squeezed to the shape of a fan, allowing propagation of signals along the direction of $\pm\vec{E}$ at speed $|\vec{E}| \leq 1$. This shows that there are perturbative degrees of freedom propagating along electric flux lines. Such causal behavior appears to be a very general feature of tachyon effective lagrangian with runway potentials. We speculate on how this may be connected to appearance of fundamental strings.

Keywords: D-branes, Tachyon Condensation.

Contents

1. Effective field theory of open string tachyon condensation

The decay of unstable D-branes [1] has served as an important laboratory in understanding the off-shell physics of open string theory. One of the more surprising aspect of this story is how powerful the tree-level theory is in reproducing the desired feature of the process [2]. For example, the so-called Boundary String Field Theory (BSFT) [3, 4] has reproduced the exact height of the tachyon potential as well as exact tension of some of the lower dimensional D-branes as solitons [5, 6]. This sort of result seems pretty surprising, since the effective field theory one employs is really only justified at the initial stage of the process near the top of the tachyon potential. We might expect that the effective field theory description of the process would be useless near the bottom of the potential, where open string degrees of freedom, or field content thereof, does not make much sense.

One hint that the use of classical effective field theory is not perhaps completely unjustified, is the fact that the field theory seems to lack most of the perturbative degrees of freedom. A toy example of this was given recently by A. Sen [7] where he considers a lagrangian of type

$$-V(T)\sqrt{-\det(\eta+\partial T\partial T)}=-V(T)\sqrt{1+(\partial T)^2}\,. \tag{1.1}$$

This form of action was first proposed by Garousi and also by Bergsheoff et al. [8] and explored previously in refs. [9, 10]. With the runaway form of potential $V\sim e^{-T}$ or e^{-T^2}, which is also natural in the BSFT type action, Sen notes that the perturbative, planewave-like solutions are completely absent as the system approaches the vacuum, $T=\infty$. At the bottom of the potential, the D-brane should have disappeared, and thus no open string degrees of freedom should be possible. Whatever the weaknesses of the effective field theory there might be, it nevertheless captures some of the crudest aspect of the tachyon condensation. Much of this depends crucially on the unusual nature of the vacuum, which requires the tachyon field to evolve in time as $\dot{T}=1$.

On the other hand, a different sort of behavior had been found in ref. [10] which considered an effective field theory with lagrangian [11]

$$-V(T)\sqrt{-\det(\eta+F)}\,, \tag{1.2}$$

where one takes T as non-dynamical and instead concentrate on the worldvolume gauge field. With this system, the counterpart of $\dot{T}=1$ above is that the electric field goes critical $|\vec{E}|=1$.

The dynamics of this system was found to be that of string-like fluid, consisting of remnant of electric flux lines, rendered pressureless in the limit of $V\to 0$. Even though the fluid is pressureless, similar to the tachyon matter, one crucial difference is that the basic object is one-dimensional and small fluctuation of such one-dimensional object is still allowed. Thus, the dynamics is not completely trivial and perturbative degrees of freedom survive. In fact, an intriguing aspect of this dynamics was that it contains a special subclass of solutions that resembles relativistic Nambu-Goto string [10, 12], allowing the wishful thought that fundamental, closed string might emerge naturally from some effective field theory of tachyon condensation. More details about string fluids can be found in [23]. Over the years, the issue of closed strings has proved to be one of most difficult question to answer in the open string tachyon condensation [10, 13, 14, 15].

In this paper, we would like to explore a general class of effective field theories of the form,

$$-V(T)\sqrt{-\det(\eta+F)}\;\mathcal{F}(z)\,, \tag{1.3}$$

with

$$z\equiv((\eta+F)^{-1})^{\mu\nu}\;\partial_\mu T\partial_\nu T\,. \tag{1.4}$$

With $\mathcal{F}(z)=\sqrt{1+z}$, this includes the above two cases as special limits, while with yet another choice of $\mathcal{F}$, the effective action is exactly the one found in the BSFT approach. In this general class of action with both gauge field and tachyon field present, the vacuum condition is modified to $\vec{E}^2 + \dot{T}^2 = 1$, which interpolates between the two extreme cases above. The main purpose of this paper is to explore this class of effective field theory and ask what kind of classical dynamics it possesses.

The notion of the effective metric proves to be quite useful in this endeavor. In non-linear theories, the causal structure of signal propagation does not coincide with that of the background spacetime (or worldvolume). Rather one must find the effective metric by expanding the action or the field equation. A typical behavior we find is that such an effective metric tends to collapse to a singular form, and restricts possible propagation of signal to some extreme forms. In the purely tachyonic case above, this collapse is isotropic and effectively sets "the speed of light" to zero, as we will see shortly. This singular limit, which can be thought of as a limit of the lorentzian spacetime, opposite to the galilean limit, is known as carrollian. This collapse of the causal structure is precisely what is responsible for disappearance of perturbative degrees of freedom.

With gauge field turned on, however, the story changes qualitatively. While the light-cone of small fluctuation still collapses, it does leave some directions open for propagation of signal; Instead of the isotropic collapse of the effective metric, one finds that the causal structure goes carrollian in all spatial directions except one, chosen by the background electric field, $\vec{E}$. Along this special direction only, small fluctuations propagate at speed $|\vec{E}|$. The bulk of this paper will be devoted to derivation of this fact, and also to its implication to tachyon condensation.

In section 2, we briefly recall the Carroll limit as a limit of vanishing speed of light. In section 3, we derive the effective metric of the purely tachyonic system, and demonstrates that the lightcone collapses completely and the effective metric is carrollian. In section 4, we explore the effective field theory with gauge fields included, and demonstrate that some perturbative degrees of freedom remain and propagates along one special direction. In section 5, we employ the hamiltonian formalism and consider the dynamics of small fluctuation as well as that of electric flux lines. For the sake of simplicity we restrict to the case of the action of ref. [8] in this section. In particular, we study how the dynamics of the fundamental string flux lines are affected by the singular causal structure. In section 6, we conclude with summary and directions for future research.

2. Carroll group

The Lorentz group SO(3,1) and its extension by spacetime translations, the Poincaré group E(3,1), are central to our understanding of issues such as causality in quantum field theory. In fact there is not a single Lorentz group but one for each value of the absolute velocity c. Of course they are all isomorphic except in the Inönü-Wigner limiting cases when $c \uparrow \infty$ or $\downarrow 0$. The former case corresponds to the Galilei group when we have instantaneous propagation and action at a distance with field satisfying elliptic partial differential equations, the latter, which is less well known, is called the Carroll group [16, 17] and corresponds to the case of no propagation at all. Fields at each spatial point evolve independently and are typically governed by ordinary differential equations with respect to just the time variable. For that reason, this case often arises as the symmetry group of an approximation scheme in which spatial derivatives are ignored compared with time derivatives. Such approximation schemes are sometimes called "velocity dominated".

Geometrically the Galilei group arises when the future light cone flattens out to become a spacelike hyperplane. The Carroll group arises when it collapse down to a timelike half line. In the galilean case only the contravariant metric tensor has a well defined limit as $c \uparrow \infty$:

$$\eta^{\mu\nu} \to \mathrm{diag}(0, 1, 1, \dots, 1) \tag{2.1}$$

and the limiting spacetime structure is called a Newton-Cartan spacetime. In the case of the carrollian limit it is the covariant metric tensor which survives

$$\eta_{\mu\nu} \to \mathrm{diag}(0, 1, 1, \dots, 1)\,, \tag{2.2}$$

and one has a carrollian spacetime.

A striking feature of the Carroll group is that the contracted Lie algebra contains a Heisenberg sub-algebra in which energy P_0 appears as a central charge. In other words, if P_i generate the spatial translations and B_i the boosts, then the only non-vanishing commutator among these 7 generators is

$$\Big[P_i, B_j\Big] = \delta_{ij} P_0\,. \tag{2.3}$$

Of course the spatial rotations J_{ij} act in the usual way. In four spacetime dimensions, there are two Casimirs, the energy P_0 and $(P_0 \vec{J} + \vec{P} \times \vec{B})^2$.

Although it emerges naturally in any classification of possible kinematic groups [18, 19], as of now, the Carroll group has played a rather minor role in physics. In this paper we explore the possibility that it enters in an essential way into the phenomenon of tachyon condensates in string theory. If the latter turn out to have a cosmological role [20, 21], then the Carroll group will certainly come into its own sometime in the future.

To illustrate how a field theory may behave in a carrollian limit, let us consider a Maxwell field in 3+1 dimensions. The standard Lorentz-invariant Maxwell equations are, after inserting the velocity of light c,

$$\frac{\partial \vec{E}}{\partial t} = c^2\,\mathrm{curl}\,\vec{B}\,, \qquad \frac{\partial \vec{B}}{\partial t} = -\,\mathrm{curl}\,\vec{E}\,, \tag{2.4}$$

and

$$\mathrm{div}\,\vec{E} = 0\,, \qquad \mathrm{div}\,\vec{B} = 0\,. \tag{2.5}$$

If we now take the limit $c \downarrow 0$, we get

$$\frac{\partial \vec{E}}{\partial t} = 0 \qquad \frac{\partial \vec{B}}{\partial t} = -\,\mathrm{curl}\,\vec{E}\,, \tag{2.6}$$

Thus in the carrollian limit the electric field $\vec{E}$ is time independent

$$\vec{E}(\vec{x}, t) = \vec{E}(\vec{x}, 0)\,. \tag{2.7}$$

JHEP09(2002)061

By contrast, the magnetic field $\vec{B}$ evolves linearly in time:

$$\vec{B}(\vec{x},t) = \vec{B}(\vec{x},0) - t \text{ curl } \vec{E}(\vec{x},0) \,. \tag{2.8}$$

In a later section, we will encounter a similarly degenerate form of gauge field theory, which differs from this example in two important aspects. One is that it derives from the nonlinear Born-Infeld action, and the other is that its causal behavior will be slightly different from this carrollian limit, in that along some special direction, set by the background "electric field" $\vec{E}$, the effective speed of light remains finite. See section 4 and 5 for more detail.

3. Effective metric for tachyon fluctuation

3.1 Open and closed string metrics

In string theory there are closed strings states and open string states. The former include the graviton and their propagation is governed by the Einstein metric $g_{\mu\nu}$ or the conformally rescaled string metric. Both have the same light cone and both define the same local Lorentz group. It seems that all closed string states couple to the same metric and hence have the same causal behavior. In other words a form of the usual equivalence principle holds for closed string states. Because closed string states couple to the dilaton the full Einstein Equivalence principle will not hold if the dilaton remains massless.

By contrast, open string states are governed by a different metric, the open string metric $G^{\rm open}{}_{\mu\nu}$. In the case of D-branes described by a Dirac-Born-Infeld action for example the open string metric differs from the induced string metric ${}_\star g_{\mu\nu}$ by terms which depend upon the gauge-invariant combination $\mathcal{F} = F_{\mu\nu} - {}_\star B_{\mu\nu}$, where $F_{\mu\nu}$ is the Born-Infeld field strength and ${}_\star B_{\mu\nu}$ is the Neveu-Schwarz potential pulled back to the brane world volume. It seems that all open string states propagating along the brane also satisfy a form of the equivalence principle, their causal properties are all governed by the light cones of the open string metric $G_{\mu\nu}$. Thus in addition to the closed string local Lorentz group there is an open string local Lorentz group. How are the two light cones related? One finds that in the Born-Infeld case, although they may coincide, the open string light cones never lie outside the closed string light cones [22, 23, 24]. The same holds true for the M-theory 5-brane [25].

The comparisons made above are between signals, both of which propagate in the brane. A different question, which arises in the AdS/CFT correspondence is the time delay suffered by signals travelling through the bulk compared with those restricted to the boundary. It seems that general principles dictate that the signal travelling in the bulk should indeed always be delayed rather than advanced [26, 27]. The distinction between bulk and brane disappears in the case of space-filling branes and in this paper we shall ignore it.

JHEP09(2002)061

3.2 Effective metric for tachyon fields

Now if considers a classical tachyon field $T(x)$ there is the possibility of yet another metric arising, or perhaps one should think of the tachyon as modifying the open string metric. The general Tachyon lagrangian density $\mathcal{L}$ in the absence of Born-Infeld etc is of the form

$$\mathcal{L} = \sqrt{-g}\, L(T, y)\,, \tag{3.1}$$

where

$$y = g^{\mu\nu}\partial_\mu T \partial_\nu T\,, \tag{3.2}$$

and as usual $g = \det\, g_{\mu\nu}$.

Thus for example Sen chooses [7]

$$\mathcal{L} = -\sqrt{-g}\, V(T)\sqrt{1+y} = -V(T)\sqrt{-\det(g_{\mu\nu} + \partial_\mu T \partial_\nu T)}\,, \tag{3.3}$$

where $V(T)$ is the tachyon potential. However other lagrangians have been considered and in any case the classical lagrangian is scheme dependent. For example the BSFT action for a non-BPS D-brane in superstring theory is, in the absence of the Born-Infeld field, [6]

$$\mathcal{L} = -\sqrt{-g}\, e^{-\frac{1}{4}T^2}\mathcal{F}(y)\,, \tag{3.4}$$

where

$$\mathcal{F}(y) = \frac{y 4^y \Gamma(y)^2}{2\Gamma(2y)}\,. \tag{3.5}$$

In the lagrangians we omit the overall tension T_p of the non-BPS Dp-brane, and we work in the unit $\alpha' = 2$.

Therefore in the sequel we shall consider the general case. The energy momentum tensor is given by

$$T^{\mu\nu} = Lg^{\mu\nu} - 2L_y \nabla^\mu T\, \nabla^\nu T\,, \tag{3.6}$$

where $\nabla^\mu T = g^{\mu\nu}\partial_\nu T$ and the subscript y denotes partial differentiation with respect to y. If T is just a function of time, $y = -\dot{T}^2$ and as it rolls, the tachyon energy density ρ and pressure P are given by

$$\rho = 2yL_y - L\,, \qquad P = L\,. \tag{3.7}$$

The weak energy condition, that is $T^{\mu\nu}p_\mu p_\nu \geq 0$ for all co-vectors p_μ which are causal with respect to the metric $g_{\mu\nu}$ will hold if $2yL_y - L \geq 0$. The Dominant Energy Condition, that is $T^{\mu\nu}p_\mu$ is future directed timelike or null with respect to the metric $g_{\mu\nu}$ for all future directed timelike or lightlike covectors p_μ will hold if $|L| \leq 2yL_y - L$. The Dominant Energy condition guarantees a degree of causal behavior in that, as originally proved by Hawking [28] (see also refs. [23, 29]), if the tachyon energy density vanishes outside some compact set, then it vanishes outside the future of that set. In other words an advancing front of energy density advances into vacuum at a speed no faster than that of light. A rather different question is how fluctuations around a tachyon background travel.

To answer that, recall that the tachyon equation of motion may be written as

$$(G^{-1})^{\mu\nu}\nabla_\mu \nabla_\nu T = \frac{L_T}{2L_y}\,, \tag{3.8}$$

where

$$(G^{-1})^{\mu\nu} = g^{\mu\nu} + \left(\frac{2L_{yy}}{L_y}\right) \nabla^\mu T \, \nabla^\nu T \tag{3.9}$$

is the inverse of the metric

$$G_{\mu\nu} = g_{\mu\nu} - \left(\frac{2L_{yy}}{L_y + 2yL_{yy}}\right) \partial_\mu T \partial_\nu T \, . \tag{3.10}$$

Evidently the characteristics of the tachyon field are given by the light cones of the metric $G_{\mu\nu}$. In other words the speed of small fluctuations around a tachyon field background is governed by the "tachyon metric" $G_{\mu\nu}$. By considering a co-vector p_μ which is lightlike with respect to the metric $g_{\mu\nu}$, that is such that $g^{\mu\nu} p_\mu p_\nu = 0$, one discovers that

$$(G^{-1})^{\mu\nu} p_\mu p_\nu = \left(\frac{2L_{yy}}{L_y}\right) (p_\mu \nabla^\mu T)^2 . \tag{3.11}$$

It follows that if $2L_{yy}/L_y$ is negative then the propagation of fluctuations of the tachyon field will be slower than light. This is actually the case for Sen's action (3.3) in the region $-1 < y$.

The energy momentum tensor and metrics for the BSFT case is somewhat complicated. It is known to satisfy both Weak Energy condition and the Dominant Energy condition. One may also check that the light cones of the metric $G_{\mu\nu}$ never lie outside the light cone of the metric $g_{\mu\nu}$. In fact, the quantity $2L_{yy}/L_y$ is always negative for $-1 < y \leq 0$ which is the region relevant for the rolling tachyon in the BSFT case [30].

3.3 The carrollian behavior of tachyonic fluctuations in rolling tachyon background

In the case of Sen's choice of tachyon action, with $\mathcal{F}(y) = \sqrt{1+y}$, one finds

$$G_{\mu\nu} = g_{\mu\nu} + \partial_\mu T \, \partial_\nu T \, , \tag{3.12}$$

and

$$(G^{-1})^{\mu\nu} = g^{\mu\nu} - \frac{1}{1+y} \nabla^\mu T \, \nabla^\nu T \, . \tag{3.13}$$

Moreover the energy momentum tensor takes the strikingly simple form

$$T^{\mu\nu} = -V(T)\sqrt{1+y}(G^{-1})^{\mu\nu} = L(G^{-1})^{\mu\nu} \, . \tag{3.14}$$

In this simplest model of Sen, the tachyon potential $V(T)$ has a minimum at infinity, and as the tachyon rolls towards it, one finds $T \to \infty$ and $|\dot{T}| \to 1$. As this happens we find that

$$G_{\mu\nu} \to \mathrm{diag}(0, 1, 1, \ldots, 1) \, . \tag{3.15}$$

Thus in the limit the metric degenerates, the tachyon light cone collapses onto a timelike half line and the tachyon fields at different spatial points are decoupled. No propagation of tachyon fluctuations can take place. This phenomenon lies at the heart of recent observations that no plane wave is possible for the tachyonic field near the bottom of the potential $T = \infty$.

In the case of the BSFT action one may use the fact that near $y=-1$ one has [30]

$$\mathcal{F}(y) \sim -\frac{1}{2(y+1)}, \tag{3.16}$$

to show that as $|\dot{T}| \to 1$, $G_{\mu\nu} \to \mathrm{diag}(0,1,\ldots,1)$. Thus we see that the BSFT metric shares with the tachyon metric $G_{\mu\nu}$ the property that it becomes carrollian near the tachyon vacuum. In general, it is obvious from the expression (3.10) that, if L_{yy} is more divergent than L_y around the rolling tachyon phase $y \sim -1$, the effective metric $G_{\mu\nu}$ becomes carrollian.

4. Born-Infeld gauge fields

If the carrollian metric governs all open string fluctuations then these also will cease to propagate. The main objective of this section is to show that this is not the case, once we consider gauge fields. The gauge field enters the action through a Born-Infeld piece, and one obvious generalization of Sen's model is[1]

$$\mathcal{L} = -V(T)\sqrt{-\det(g+F+\partial T\partial T)}\,. \tag{4.1}$$

In fact, this form of action had been proposed in ref. [8] and studied in much detail in ref. [10]. To allow uniform treatment of this action and the BSFT action, we rewrite this action as

$$\mathcal{L} = -V(T)\sqrt{-\det(g+F)}\mathcal{F}(z)\,, \tag{4.2}$$

with $\mathcal{F} = \sqrt{1+z}$, where

$$z = \left((g+F)^{-1}\right)^{(\mu\nu)}\partial_\mu T\partial_\nu T\,. \tag{4.3}$$

Note that only the symmetric part of $(g+F)^{-1}$ enters the expression of kinetic term for T.[2] Then, the BSFT action can be written in the same form with a different $\mathcal{F}$, as given in (3.5).

The main difference one encounters when the gauge field in included is that vacuum structure changes substantially. It was observed in ref. [10] that, for Sen's effective action the determinant part

$$\sqrt{-\det(g+F+\partial T\partial T)} \tag{4.4}$$

itself should vanish as $V \to 0$. This is really why $\dot{T}=1$ in a purely tachyonic system, while in ref. [10], the vacuum considered is such that $\dot{T}=0$ and $E=1$ instead. In is clear that these two extreme case are connected by a continuous family of vacua

$$\dot{T}^2 + E^2 = 1\,. \tag{4.5}$$

As will be shown shortly, this holds not only for the above form of the action, but also for the effective action derived from BSFT method.[3]

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[1] Rolling tachyon coupled to gauge fields has been investigated also in refs. [31, 32].

[2] This metric $\widetilde{g}$ is what one usually calls "open string co-metric," for instance, in noncommutative setting. In the current context, with $\dot{T}$ nonzero, this metric is no longer "open string metric".

[3] The tachyon effective action proposed by Lambert and Sachs [33] does not possess this property.

In this section, we will investigate how fluctuation of the tachyon field and that of the gauge field behave in generic vacuum with both $\dot{T}$ and E nontrivial. A single gauge field in $p+1$ dimensions possesses $p-1$ propagating degrees of freedom, due to the gauge invariance, while T would give, at least in ordinary circumstances, one degree of freedom. In a nutshell, the effective metric for these $(p-1)+1=p$ fluctuation is no longer carrollian but given by

$$G_{\mu\nu} \to \lim_{\epsilon\to 0} \mathrm{diag}(-\epsilon, \epsilon/E^2, 1, \ldots, 1)\,, \tag{4.6}$$

as the tachyon condensation proceeds. This causal structure is different from the carrollian limit, since along one spatial direction (chosen by the direction of the background electric field E) gauge fluctuations may propagate. Interestingly the speed at which the signal propagates is precisely E.[4]

4.1 Background equations of motion

First, let us show that the constant electric field is allowed in the rolling tachyon phase, as a classical solution of the tachyon system coupled to gauge field. We shall use the following general action which includes both Sen's action and the BSFT action,

$$\mathcal{L} = -V(T)\sqrt{-\det(\eta+F)}\;\mathcal{F}(z)\,. \tag{4.7}$$

Recall that Sen's action is given by $\mathcal{F}(z)=\sqrt{1+z}$, while this $\mathcal{F}(z)$ for the BSFT action is a complicated function (3.5) whose singular behavior around $z\sim -1$ is given by (3.16). Now we turn on the electric field only along x^1, and assume that all the fields are homogeneous: the electric field and the tachyon depend only on time. Then the lagrangian reduces to

$$\mathcal{L} = -V(T)\sqrt{1-E^2}\mathcal{F}\left(\frac{-\dot{T}^2}{1-E^2}\right). \tag{4.8}$$

The equations of motion are given by

$$\dot{\pi}=0\,, \qquad \dot{P}+V'\sqrt{1-E^2}\mathcal{F}=0\,, \tag{4.9}$$

in which we have defined the conjugate momenta as

$$\pi \equiv \frac{\delta\mathcal{L}}{\delta E} = V\frac{E}{\sqrt{1-E^2}}\left(\mathcal{F}+\frac{2\dot{T}^2}{1-E^2}\mathcal{F}'\right), \qquad P\equiv\frac{\delta\mathcal{L}}{\delta\dot{T}} = 2V\frac{\dot{T}}{\sqrt{1-E^2}}\mathcal{F}'\,. \tag{4.10}$$

Note that π is the electric induction which is often denoted by D. The first equation in (4.9) shows that the electric induction or flux π is a conserved quantity. Another conserved quantity is the energy hamiltonian which is given by

$$\mathcal{H} = \pi E + P\dot{T} - \mathcal{L} = V\frac{1}{\sqrt{1-E^2}}\left(\mathcal{F}+\frac{2\dot{T}^2}{1-E^2}\mathcal{F}'\right). \tag{4.11}$$

[4] One might be mislead to think that this actually means gauge fluctuation is absent, since in 1+1 dimensions gauge degrees of freedom are absent. We emphasize that this is not the case here. While propagation may be allowed only along a specific direction, the gauge field still lives on entire worldvolume of the brane. Nontrivial degrees of freedom comes from transverse distributions of electric flux lines.

From these expression, one observes an interesting relation

$$\pi = E\mathcal{H}\,. \tag{4.12}$$

Since both π and $\mathcal{H}$ are conserved quantities, the constant E is a consistent solution of the system.

In particular, If we choose the initial condition for the rolling as

$$T\mid_{t=0} = T_0\,, \qquad \dot{T}\mid_{t=0} = 0\,, \tag{4.13}$$

where T_0 is a positive constant, then the conserved energy can be evaluated at $t=0$ as

$$\mathcal{H} = \frac{V(T_0)\mathcal{F}(0)}{\sqrt{1-E^2}}\,. \tag{4.14}$$

Therefore with the above relation (4.12) we have

$$E = \frac{\pi}{\sqrt{V(T_0)^2\mathcal{F}(0)^2 + \pi^2}}\,. \tag{4.15}$$

This means that the infinite condensation of the electric flux $\pi = \infty$ is equivalent to the limit $E = 1$.

As the tachyon rolls down the potential hill $V(T)$, it approaches the true vacuum at which the potential vanishes, $V(T=\infty) = 0$. Therefore, it is obvious from the expression for π (4.10) and $\mathcal{H}$ (4.11) that the combination

$$\left(\mathcal{F} + \frac{2\dot{T}^2}{1-E^2}\mathcal{F}'\right) \tag{4.16}$$

should be divergent, to maintain the constancy of π and $\mathcal{H}$. For both Sen's action and the BSFT action, this divergence is achieved when the argument z of the function $\mathcal{F}$ approaches -1 as in the case of the rolling tachyon without the electric field. The equation $z=-1$ gives the rolling tachyon solution with a constant electric field,

$$\dot{T}^2 + E^2 = 1\,. \tag{4.17}$$

It follows from this expression that the critical electric field $E = 1$ is a special case: in this limit the flux π diverges and the physics may be dominated by the fundamental string. The tachyon does not roll, and the situation is similar to that of the supertubes [34]. In the supertube configurations, tachyons caused by a brane-antibrane configuration become massless and the whole system is stable due to the critical limit of the electric field.

4.2 Unidirectional propagation of small fluctuations

After having gone through previous section, one might expect that the only change as far as the tachyon fluctuation goes, is that $g^{\mu\nu}$ is replaced by $(\widetilde{g})^{-1}$ which is the symmetric part of $(g+F)^{-1}$. For Sen's action, this observation would give to the effective metric

$$G_{\mu\nu} = \widetilde{g}_{\mu\nu} + \partial_\mu T\,\partial_\nu T \to \mathrm{diag}(0, 1-E^2, 1, \ldots, 1)\,. \tag{4.18}$$

which is again carrollian for $E \neq 0$. However, this naive expectation is incorrect: the gauge fluctuations and the tachyon fluctuation do not in general separate nicely. Rather they are mixed together so that we have to consider them simultaneously. In the following, we show that the tachyon and gauge fluctuations are mixed and all $(D-1)$ degrees of freedom propagate, along a particular direction determined by the background electric field.

First in order to illustrate the point, let us analyze Sen's action. In this action, the kinetic term for the tachyon T appears in the square root as if it were one of the transverse scalar field. The transverse scalar field in the Dirac-Born-Infeld action can be treated as one of the additional gauge field [35] through the T-duality along the transverse direction. Then this means that we can regard T as a transverse scalar field except that it experiences the potential $V(T)$. However, this potential term is irrelevant for our analysis, since we need only the causal behavior which is encoded in the kinetic term. The only information which we need from the potential term is that we are going to the rolling tachyon limit $z \to -1$. Therefore, for our purpose, it is sufficient to consider the Born-Infeld equations of motion in one higher dimensions $p+2$:

$$\left(\frac{1}{\eta+F}\right)^{(\mu\nu)} \partial_\mu F_{\nu\rho} = 0\,. \tag{4.19}$$

Here note that the gauge field strength is that in $p+2$ dimensions. The indices are $\mu, \nu, \rho = 0, 1, \cdots, p, T$, and

$$F_{\mu T} \equiv \partial_\mu T\,. \tag{4.20}$$

We use the above equations of motion with the understanding that $\partial_T = 0$.

Let us define a set of new worldvolume coordinates which turn out to be useful,

$$\begin{pmatrix} \widetilde{x}^T \\ \widetilde{x}^1 \end{pmatrix} \equiv \frac{1}{\dot{T}^2 + E^2} \begin{pmatrix} \dot{T} & E \\ -E & \dot{T} \end{pmatrix} \begin{pmatrix} x^T \\ x^1 \end{pmatrix}. \tag{4.21}$$

This redefinition is the "target space rotation" which was investigated in ref. [36]. The origin of this rotation is as follows. First, let us take a different T-duality, not along the transverse direction but along x^1. Then the world volume is p dimensional, and we have two transverse scalar fields, T and A_1. In this T-dualized language, the rolling tachyon limit

$$\dot{T} \to \sqrt{1-E^2} \tag{4.22}$$

can be understood as a light-like limit of the worldvolume, i.e. the $(p-1)$-brane is travelling with the speed of light along the direction $(x^1, x^T) = (E, \sqrt{1-E^2})$. Our target space rotation (4.21) is just the rotation to orient the brane motion into the direction along $\widetilde{x}^T$.

It is known that when the worldvolume is lightlike then the induced metric on the brane is carrollian. In our case, we have to take a T-duality back to have $p+1$ dimensional world volume, thus the question is not so simple. Actually, we will find that the dynamics in the rolling tachyon limit is not carrollian: a part of the gauge fields become dynamical and can propagate along x^1, with the effective metric (4.6).[5]

[5] A simple reason may be that the last T-duality is not orthogonal to the direction of the motion of $(p-1)$-brane. It induces the electric field back [37] and non-trivial dependence on the T-dualized direction (x^1).

JHEP09(2002)061

By the target space rotation, the gauge field strength is transformed as usual,

$$\widetilde{F}_{\tilde{\mu}\tilde{\nu}} = \frac{\partial x^\rho}{\partial \widetilde{x}^\mu} \frac{\partial x^\sigma}{\partial \widetilde{x}^\nu} F_{\rho\sigma} \,, \tag{4.23}$$

therefore the tensor appearing in (4.19) described by the new coordinates becomes

$$\begin{aligned} \left(\frac{1}{\eta + \widetilde{F}}\right)_{\rm sym} &= \begin{pmatrix} -1 & 0 & \cdots & 0 & E^2 + \dot{T}^2 \\ 0 & & \cdots & & 0 \\ -E^2 - \dot{T}^2 & 0 & \cdots & 0 & 1 \end{pmatrix}^{-1}_{\rm sym} \\ &= \mathrm{diag}\left(\frac{-1}{1 - E^2 - \dot{T}^2}, 1, \cdots, 1, \frac{1}{1 - E^2 - \dot{T}^2}\right). \end{aligned} \tag{4.24}$$

In the limit $\dot{T} \to \sqrt{1 - E^2}$ the equations of motion (4.19) reduces to the two dimensional one,

$$\partial_0 \widetilde{F}_{0\widetilde{T}} - \partial_{\widetilde{T}} \widetilde{F}_{\widetilde{T}\widetilde{\mu}} = 0 \,. \tag{4.25}$$

Since we are studying the fluctuation around the rolling tachyon background, we expand the fields as

$$T = T_0 + t \,, \qquad F_{\mu\nu} = E(\delta_{\mu 0}\delta_{\nu 1} - \delta_{\mu 1}\delta_{\nu 0}) + f_{\mu\nu} \,, \tag{4.26}$$

where $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, and t, a_μ are the fluctuations. Then the causal structure for the fluctuation can be extracted from (4.25) if one replace $\widetilde{F}$ by its fluctuation and (4.21) by its background value, since we need only the term in which the fluctuations receives two derivatives. We shall work in the gauge $a_0 = 0$.

For $\widetilde{\mu} = \widetilde{T}$, the equation (4.25) is reduced to

$$(\partial_0)^2 \left[E a_1 + \sqrt{1 - E^2}\, t\right] = 0 \,. \tag{4.27}$$

Hence if we define a linear combination

$$\hat{t} \equiv \sqrt{1 - E^2}\, t + E a_1 \,, \tag{4.28}$$

this $\hat{t}$ has no dynamics. For $\widetilde{\mu} = 0$, we obtain $E\partial_1\partial_0\hat{t} = 0$ and this is consistent with the fact that $\hat{t}$ has no dynamics.

For $\widetilde{\mu} = 2, \cdots, p$, the resultant equations of motion are

$$\left(\partial_0^2 - E^2\partial_1^2\right) a_\mu + E\partial_1\partial_\mu\hat{t} = 0 \,. \tag{4.29}$$

Therefore if we turn off the non-dynamical fluctuation $\hat{t}$, the transverse fluctuation gauge fields a_μ are subject to the 2 dimensional Klein Gordon equation which shows the propagation with the speed E. In other words, the causal structure of these fluctuations a_μ ($\mu = 2, \cdots, p$) is determined by the effective metric (4.6).

We have seen that a combination of a_1 and t (4.28) is non-dynamical. What about another combination which is orthogonal to that? The equation for $\widetilde{\mu} = 1$ is

$$-E\partial_0^2 t + \sqrt{1 - E^2}\partial_0^2 a_1 + E^2\partial_1^2 t = 0 \,. \tag{4.30}$$

Using (4.27), the above equation is reduced to

$$\left(\partial_0^2 - E^2\partial_1^2\right) t = 0\,. \tag{4.31}$$

Therefore another combination also is subject to the effective metric (4.6).

After all, there are p dynamical degrees of freedom which propagates along x^1 with the speed E.

4.3 Fluctuations and general BSFT type action

The analysis in section 4.2 took full advantage of regarding the tachyon as a transverse scalar field. In this subsection, we show that the result above is quite universal, and does not refer to any concrete form of the kinetic term function $\mathcal{F}(z)$ in the lagrangian. We utilize only the behavior of $\mathcal{F}(z)$ in the rolling tachyon limit $z \to -1$.

We use an alternative method of expanding the effective action itself. This is basically equivalent to analyzing the equations of motion as in section 4.2. We expand the lagrangian (4.7), and collect all the terms quadratic in the fluctuation. For our purpose it is clear that we don't need the expansion of $V(T)$, since the expansion of the potential term does not give derivatives of fluctuations and thus it is not related to the causal structure which we want. After a straightforward calculation, we obtain the expansion of $\mathcal{F}$ and the Born-Infeld term $\sqrt{-\det(\eta+F)}$ as

$$\mathcal{L} = -V(T_0)\sqrt{1-E^2}\left[\mathcal{F}''(z^{(0)})\mathcal{L}^{(2)} + \mathcal{F}'(z^{(0)})\mathcal{L}^{(1)} + \mathcal{F}(z^{(0)})\mathcal{L}^{(0)}\right], \tag{4.32}$$

where $z^{(0)}$ is the background value of $z \equiv \widetilde{g}^{\mu\nu}\partial_\mu T \partial_\nu T$,

$$z^{(0)} = \frac{-\dot{T}_0^2}{1-E^2}\,, \tag{4.33}$$

and the components $\mathcal{L}^{(i)}$ $(n = 0,1,2)$ have the coefficient $(\partial_z)^n \mathcal{F}(z)|_{z=z^{(0)}}$ for each. The definition of them are as follows:

$$\mathcal{L}^{(2)}(t,a) \equiv \frac{2\dot{T}_0^2}{(1-E^2)^2}\left(\partial_0 t + \frac{\dot{T}_0 E}{1-E^2} f_{01}\right)^2, \tag{4.34}$$

$$\begin{aligned}\mathcal{L}^{(1)}(t,a) \equiv\ & \left(\frac{-1}{1-E^2}(\partial_0 t)^2 + \frac{1}{1-E^2}(\partial_1 t)^2 + (\partial_i t)^2\right) + \\ & + \left(\frac{2E\dot{T}_0}{(1-E^2)^2} f_{10}\partial_0 t + \frac{2E\dot{T}_0}{1-E^2} f_{i1}\partial_i t\right) + \\ & + \left(\frac{-(1+E^2)\dot{T}_0^2}{(1-E^2)^3}(f_{01})^2 + \frac{-\dot{T}_0^2}{(1-E^2)^2}(f_{0i})^2 + \frac{E^2\dot{T}_0^2}{(1-E^2)^2}(f_{1i})^2\right).\end{aligned} \tag{4.35}$$

Here i runs from 2 to p. The last component $\mathcal{L}^{(0)}$ includes $(f_{ij})^2$ for example, however the expression for this term is unnecessary in the following discussion.

Now we observe that the original kinetic function $\mathcal{F}(z)$ has the following behavior in the rolling tachyon limit $z \to -1$:

$$\mathcal{F}(z) \sim \mathcal{N}(1+z)^n \qquad (z \sim -1) \tag{4.36}$$

where $\mathcal{N}$ is a normalization constant, and a real number n satisfies

$$n < 1\,, \qquad n \neq 0\,. \tag{4.37}$$

This is derived from the fact that the combination (4.16) is diverging in the limit. The relation (4.37) is satisfied for both Sen's action ($n = 1/2$) and the BSFT action ($n = -1$). Then it follows that in the rolling tachyon limit

$$\left|\mathcal{F}''(z^{(0)})\right| \gg \left|\mathcal{F}'(z^{(0)})\right| \gg \left|\mathcal{F}(z^{(0)})\right|\,. \tag{4.38}$$

This means that in the expanded lagrangian, the dominant term is $\mathcal{L}^{(2)}$ whose coefficient is most rapidly diverging. If we take $a_0 = 0$ gauge, then in the rolling tachyon limit $\dot{T}_0 \to \sqrt{1-E^2}$ the expression (4.34) is arranged as

$$\mathcal{L}^{(2)} = \frac{1}{(1-E^2)^{3/2}}(\partial_0 \hat{t})^2 \tag{4.39}$$

where we have used the linear combination (4.28). This $\hat{t}$ is the mode parallel to the motion of the brane in the T-dual language which was studied in section 4.2. The lagrangian for the mode $\hat{t}$ is effectively given by (4.39) and other terms becomes irrelevant in the rolling tachyon limit. Hence $\hat{t}$ becomes non-dynamical.

The dynamics for the gauge fields a_i turns out to be different. The equation of motion for a_i derived from the fluctuation lagrangian (4.32) is

$$\frac{2\dot{T}_0^2}{(1-E^2)^2}\left(\partial_0^2 - E^2\partial_1^2\right)a_i + \frac{2E}{1-E^2}\partial_1\partial_i\hat{t} = 0\,. \tag{4.40}$$

This is precisely what we have found in the analysis of Sen's action, (4.29). Therefore if we turn off the non-dynamical field $\hat{t}$, then the gauge field a_i ($i = 2, \ldots, p$) is subject to the effective metric (4.6).

It is straightforward to see from the action (4.35), that another linear combination of t and a_1 which is orthogonal to $\hat{t}$ is subject again to the effective metric (4.6) if we turn off the non-dynamical $\hat{t}$. This coincides with the analysis in section 4.2.

These results turn out to be quite universal, as follows from the simple limit behavior of the kinetic term of the action, (4.38).

5. Hamiltonian dynamics and string-like objects

Now that we have explored small fluctuations around homogeneous vacua, let us turn to dynamics of flux lines. The question of gauge dynamics upon tachyon condensation has an interesting history in recent years. Tachyon condensation here is a process of unstable branes decaying, seen from open string degrees of freedom. Of various confusions about this process, perhaps the most intriguing is what happens to electric fluxes on the worldvolume of the decaying brane [13, 14, 38]. Electric flux lines of the Born-Infeld action carry fundamental string charges from viewpoint of the spacetime, which is a conserved charge.

While one might be content to say that the violent annihilation process pushes all fluxes away to spatial infinity, just as most of worldvolume energy is dispersed, but this is not quite satisfactory. For instance, one can imagine setting up a device which selectively keeps a particular conserved charge. Something must be left behind and carry that particular conserved charge, and we should then ask what object carries the leftover charge. The obvious answer to this, in case of fundamental string charge in the form of electric flux lines, is of course the fundamental string itself [13]. Less clear is precisely how this formation of fundamental string from electric flux line happens. A nonperturbative confinement mechanism has been proposed, while some advocate a classical, still unknown, origin of such phenomenon.

Ref. [10] by two of the authors explored this question from purely classical viewpoint. The exact dynamics of Sen's effective action contains a fluid of flux lines at the vacuum, each of which behaves remarkably like an infinitesimal version of Nambu-Goto string. What is absent in the classical dynamics is precisely something that will bind such flux lines into a string-like objects of small width. In fact, a classically exact scale invariance was shown to exist and implies that flux lines have no transverse pressure whatsoever. On the other hand, exactly because of this lack of pressure, the possibility opens up that the slightest quantum correction could break the scale invariance and drive the flux fluid into quantized units of flux bundles, which may then be identified with fundamental strings.

In any case, by an ansatz within the classical dynamics, one could still consider tightly bunched flux lines, which thanks to the pressureless nature of the fluid, are of finite energy per length. When such a flux bundle is squeezed into the form of delta function distribution, the resulting string-like configuration follows the classical Nambu-Goto dynamics [10, 39].[6] In this section, we would like to discuss how rolling tachyon background and the tachyon fluctuation figure into dynamics of flux lines.

5.1 Small fluctuations in hamiltonian viewpoint

The derivation of hamiltonian in ref. [10] may be applied here straightforwardly, since the tachyon kinetic terms enters as if it were one of transverse scalars. The hamiltonian of the combined system of tachyon and gauge field is,

$$\mathcal{H} = \sqrt{\pi^i\pi^i + P^2 + (\pi^i\partial_i T)^2 + (F_{ij}\pi^j + \partial_i T\, P)^2 + V^2\det(h)}\,, \tag{5.1}$$

where $h_{ij} \equiv \eta_{ij} + F_{ij} + \partial_i T \partial_j T$ with Roman indices i, j running over all spatial directions, $1, 2, \ldots, p$. The vector π_i is the conjugate momenta of the gauge field, which corresponds to the conserved electric flux, or equivalently the fundamental string density. Similarly P is the conjugate momentum of T. In the limit of tachyon condensation $V \to 0$, we find a much simplified hamiltonian,

$$\mathcal{H} = \sqrt{\pi^i\pi^i + P^2 + (\pi^i\partial_i T)^2 + (F_{ij}\pi^j + \partial_i TP)^2}\,, \tag{5.2}$$

from which all dynamics, in principle, can be read off.

There is a simpler way to express this hamiltonian. We invent a new direction and pretend that T is a component of gauge field along this new imagined direction:

$$A_T = T \tag{5.3}$$

[6] This feature turns out to extend to directions orthogonal to the brane worldvolume, as was later demonstrated by A. Sen [12].

and similarly $\pi^T = P$. Then the hamiltonian is succinctly written as

$$\mathcal{H} = \sqrt{\pi^M \pi^M + (F_{MK}\pi^K)^2}\,, \tag{5.4}$$

with the understanding that $\partial_T \equiv 0$ on any object. Indices M, K runs over $1, 2, \ldots, p$ and T. Hamiltonian equations of motion are then,

$$\begin{aligned} \dot{A}_M &= \frac{1}{\mathcal{H}}\left(\pi_M + F_{LK}\pi^K F_{LM}\right), \\ \dot{\pi}^M &= \partial_L\left(\frac{1}{\mathcal{H}}\left(\pi^M F_{LK}\pi^K - \pi^L F_{MK}\pi^K\right)\right). \end{aligned} \tag{5.5}$$

Let us expand

$$\begin{aligned} \pi^M &= (1+\epsilon)\pi_0^M + \delta\pi^M, \\ F_{MK} &= \delta F_{MK}\,, \end{aligned} \tag{5.6}$$

around a constant and uniform π_0^M background, which could consist of any combination of $\dot{T}$ and E. We define $\delta\pi^M$ to be orthogonal to the background value π_0^M, and encode the rest in ϵ. Taking antisymmetric derivative of $\dot{A}_M$ equation, and truncating to the first order perturbation, we find

$$\delta\dot{F}_{MK} = \frac{1}{\mathcal{H}_0}\left(\partial_M\delta\pi_K - \partial_K\delta\pi_M\right). \tag{5.7}$$

Taking time derivative of the π equation and inserting this relationship,

$$\ddot{\epsilon} = -\frac{1}{\mathcal{H}_0^2}(\pi_0^K\partial_K)(\partial_L\delta\pi^L)\,, \tag{5.8}$$

which shows that $\epsilon\pi_0^M$, one linear combinations of A_i and T, is not dynamical. Rather its evolution is determined completely by that of $\delta\pi^M$. This may be regarded as a consequence of Gauss's constraint, even though it involves fluctuation of T which is not charged. The orthogonal part, on the other hand, obeys a dynamical equation

$$\delta\ddot{\pi}^M = \frac{1}{\mathcal{H}_0^2}(\pi_0^K\partial_K)^2\delta\pi^M\,, \tag{5.9}$$

which are 1+1 dimensional Klein-Gordon equations.

Recall that $\partial_T \equiv 0$ by definition. Labelling the direction of $\vec{E}$ (thus that of $\vec{\pi}$) as 1, we have

$$\left(\partial_0^2 - E^2\partial_1^2\right)\delta\pi^M = 0\,, \tag{5.10}$$

the most general solution to which has the form

$$h_L(Ex^0 + x^1, x^2, x^3, \ldots, x^p) + h_R(Ex^0 - x^1, x^2, x^3, \ldots, x^p)\,. \tag{5.11}$$

This shows that, although $\delta\pi^M$ may have dependence on coordinates transverse to $\vec{E}$, such dependence does not propagate. Rather only variation along $\vec{E}\cdot\vec{x}$ may propagate with speed E. This faithfully reproduces behavior found in previous section via analysis of the effective metric. For the purely rolling tachyon limit of $E = 0$, the solution has no time-dependence at all, matching exactly onto the carrollian behavior.

Thus, in general, p degrees of freedom propagate along $\vec{E} = \vec{\pi}/\mathcal{H}$ at speed $|\vec{E}|$. This analysis can be repeated in the presence of transverse scalars X^I in exactly the same way. Again we may treat transverse scalars X^I, $I = p+1, p+2, \ldots, D-1$, as if they were gauge fields along additional directions. For an unstable brane in D spacetime dimensions, we find $D-1$ degrees of freedom propagating unidirectionally along the vector $\pm\vec{E}$ at speed $|\vec{E}|$.

5.2 String fluid in rolling tachyon background

There are a few difficulties one encounter in constructing string-like objects in this classical setting. For instance, let us consider the following class of solutions to the hamiltonian equation of motion;

$$\begin{aligned} \pi^M &= \hat{m}^M \Phi(x), \\ F_{MK}\hat{m}^K &= 0\,, \end{aligned} \tag{5.12}$$

with a constant unit vector $\hat{m}$, provided that

$$\hat{m}^M \partial_M \Phi(x) = 0\,. \tag{5.13}$$

In terms of P and $\vec{\pi}$, this means that we have a family of solutions where,

$$\begin{aligned} \pi^i &= \hat{n}^i \cos\theta\, \Phi(x), \\ P &= \sin\theta\; \Phi(x)\,, \end{aligned} \tag{5.14}$$

with

$$\hat{n}^i \partial_i \Phi = 0\,, \tag{5.15}$$

and also

$$\begin{aligned} \hat{n}^i \partial_i T &= 0\,, \\ \sin\theta\, \partial_i T &= \cos\theta\, \hat{n}^j F_{ji}\,, \end{aligned} \tag{5.16}$$

with an arbitrary angle θ. Unit vector $\hat{n}$ is related to $\hat{m}$ above by

$$\hat{m}^M = (\cos\theta\, \hat{n}^i, \sin\theta)\,. \tag{5.17}$$

It is straightforward to verify that such a configuration solves the hamiltonian equation of motion trivially.

This class of solutions, where both flux density $|\vec{\pi}|$ and the hamiltonian $\mathcal{H}$ is proportional to Φ, represents an arbitrary transverse distribution of flux lines accompanied by tachyon matter following the same pattern of distribution.

$$\begin{aligned} |\vec{\pi}| &= \cos\theta\, \Phi\,, \\ P &= \sin\theta\, \Phi\,, \\ \mathcal{H} &= \Phi\,. \end{aligned} \tag{5.18}$$

Typically there are infinitely many possible choices of Φ allowed by the constraints, so the same sort of infinite degeneracy that had plagued previous attempts at constructing tightly bound string-like objects, shows up here again. There are infinite number of possibilities for Φ with a fixed and finite total flux and thus with the same total energy.

With P approaching some constant value at infinity, we must be more careful for we wish to consider finite total flux. The above class of solution indicates that flux distribution tends to match that of P and spread out all over the space, but a more detailed analysis is required. Regardless of any such detail, however, this modification cannot help for the following reason: In the actual tachyon condensation of unstable branes, the decaying brane is coupled to closed strings and will lose energy by emission of the latter degrees of freedom. If we wish to look for fundamental string in the final vacuum state, we should set $P = 0 = \mathcal{H}$ in the asymptotic region. This brings us back to the degeneracy problem above.

5.3 Closed flux string

With possibility of rolling tachyon mixing with $\vec{E}$, on the other hand, we are able to remove one other difficulty in constructing closed-string-like object. Without the rolling tachyon, we have nowhere vanishing electric field with $|\vec{E}| = 1$, which makes it difficult to envisage a closed loop of electric flux. One ends up with singular configuration of $\vec{E}$ necessarily since closed loop of flux lines involve a winding number

$$\oint d\vec{l} \cdot \vec{E}\,, \tag{5.19}$$

which cannot be smoothly taken away unless $E = 0$ somewhere. While the singularity does not translate to divergent energy or action in this case, it is still an uncomfortable situation.

When the rolling tachyon is taken into account, the prospect looks better. Since the vacuum condition is modified to $E^2 + \dot{T}^2 = 1$, E can take any value between 0 and 1 and closed loops of flux lines are much easier to conjure. For instance, we may imagine a tightly bunched flux lines, outside of which the vacuum is dominated by rolling tachyon $\dot{T} = 1$. Since $|\vec{E}|$ is no longer restricted to be 1, the potential singularity due to the winding number is naturally resolved.

One very intriguing aspect of such configurations is that the propagation of small fluctuation is allowed only near the core where flux lines are densely populated. For instance, we could consider a very tight flux bundle, at the centre of which $|\vec{E}|$ approaches 1. This can be easily achieved by having a divergent $\vec{\pi}$ at the core. The only propagation of signals allowed are then along the $1 + 1$ dimensional core of the bundle, where signals propagate at speed of light. Outside, where $E = 0$, no small fluctuation is allowed since the causal structure became carrollian.

This way, we have conjured up a $1 + 1$ dimensional coherent object initially realized on $p + 1$ dimensional worldvolume. Furthermore, there are $D - 1$ degrees of freedom propagating only along the core of the object. The configuration could be closed or infinitely extended. This looks remarkably similar to a relativistic string, except for one problem;

it appears that classical dynamics has no mechanism to bind flux lines into such a tight bundle. This can be also seen from the fact that the number of propagating degrees of freedom is $D-1$, instead of $D-2$ we came to expect from a relativistic Nambu-Goto string; the additional degree of freedom must be related to how the flux lines are distributed over the transverse directions. We believe that there is a quantum mechanical reason for binding of flux lines, without which proper understanding of closed string is not possible [13, 14].

6. Conclusion and discussions

In this paper, we studied classical systems consisting of open string tachyon coupled to gauge field. Since the dissipative couplings to closed string degrees of freedom are suppressed, the system is conserved and the initial energy of D-branes and their worldvolume fields is retained in the form of tachyon matter and fluid of electric flux lines. Tachyon condensation is specified by a combination of rolling tachyon $\dot{T}$ and background electric field $\vec{E}$, satisfying $E^2 + \dot{T}^2 = 1$. The purely rolling tachyon background, $\dot{T} = 1$, forces a carrollian limit for all small fluctuation, and this explains the absence of perturbative degrees of freedom. On the other hand, general vacuum with $|\vec{E}| \neq 0$ allows 1+1 dimensional propagation of small fluctuations. In this case, the signal propagate at speed $|\vec{E}|$.

In section 4, we have seen that the causal structure of the gauge and tachyon fluctuations in the rolling tachyon phase is universal in that it does not depend much on the structure of the kinetic function $\mathcal{F}(z)$. What is required for the form of the function $\mathcal{F}(z)$ is only to have a rolling tachyon solution, $E^2 + \dot{T}^2 = 1$. In fact, the effective actions of non-BPS branes can be determined up to the field redefinition, which corresponds to the choice of the renormalization scheme in deriving the effective actions. For example, one may change $T \to T + a(T)z + \cdots$, as results in the change of the function $\mathcal{F}(z)$ [32]. In order to study the dynamics without referring to specific forms of the effective action, one may come back to the BSFT formulation and consider the rolling tachyon background there. In the appendix, we make a brief comment on this for a special case $E = 0$. A more general study will be necessary for studying the causal structure of the massive modes and general open string excitations in the rolling tachyon phase.

Although in this paper and most of the literature an homogeneous tachyon profile has been employed as an initial condition, it is expected that in general the tachyon may roll down inhomogeneously [40]. It was shown in ref. [41] that inhomogeneous decay of non-BPS brane may hit a singularity of localized energy in the rolling process and this can be interpreted as the formation of lower dimensional branes. If we include a background electric field in that setup, it might describe a formation of (F, D) bound states. In this respect it would be intriguing to analyse the effective metric and fluctuation dynamics in the inhomogeneous rolling tachyon background with electric fields.

In particular, one of more important outstanding questions is how inhomogeneous distribution of electric flux lines evolves in time. As we saw in section 5, there appears to be no reason to believe that fluxes would come together and form a localized string-like object. On the other hand, the causal structure found above has interesting implications once the flux lines are somehow grouped into well-separated bundles of flux strings; it tells us that the only degrees of freedom remaining would be those that propagate along the flux string, at speed of the light, and dictate how the flux string moves. Such a flux string will behave like a classical Nambu-Goto string [10]. While the classical dynamics above has one additional degree of freedom which must be related to the thickness of such a flux string, this must be lifted somehow by the (still unknown) binding mechanism.

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A. Universality of the open string metric

To generalize our result to the one valid for other open string excitations, it is useful to remember how the effective open string metric $G^{\rm open}_{\mu\nu}$ was derived in string theory. If the gauge field strength acquires a vev, the effective metric is obtained by the two point function on the boundary of the disc worldsheet with infinite number of the boundary insertion of the condensed gauge field. This effectively changes the boundary condition of a worldsheet and the Green function. The tensor coefficient of this Green function $\langle X^\mu X^\nu \rangle$ gives the effective metric.

Therefore, we consider the two point function of a special two dimensional theory with the boundary insertion corresponding to the rolling tachyon. In the BSFT formalism the kinetic term of the open string component field $f^{\mu_1,\cdots,\mu_n}$ is given by the two point function

$$\langle V(f) V(f) \rangle \,, \tag{A.1}$$

which is evaluated using the following two dimensional field theory [3]:

$$L = \frac{1}{8\pi} \int_\Sigma d^2\sigma \sqrt{h} h^{\alpha\beta} \partial_\alpha X \partial_\beta X + \frac{u}{8\pi} \oint_{\partial\Sigma} d\theta X^2 \,. \tag{A.2}$$

In the two point function, the indices of the field f are related to the X^{μ_i} operator in the vertex operators $V(f)$. The boundary Green function in this two dimensional field theory after the regularization was given in ref. [3] as

$$G(\theta, \theta') = 2 \sum_{k \in Z} \frac{1}{|k| + u} \exp(ik(\theta - \theta')) \,. \tag{A.3}$$

Here note that u should be positive in the original BSFT where the boundary interaction is for the spatial X, but in the rolling tachyon we have to consider this X as X^0 and thus u is effectively negative, with respect to the first term in the action (A.2). If u is positive, the above Green function is finite and well defined. However, if u is negative, and especially for $u = -1$, the above Green function diverges. In fact, for the rolling tachyon limit

JHEP09(2002)061

$u = -(\dot{T})^2 = -1$. Therefore, the time component of the co-metric is divergent, hence the effective metric appearing in the kinetic terms of all the open string excitations becomes carrollian in the rolling tachyon limit. This is consistent with the results in this paper, since as seen in section 4, especially when $E = 0$ all the gauge fluctuations are governed also by the carrollian metric. With more precise evaluation of the propagator (A.3), we hope that the exact form of the effective metric (3.9) may be obtained.

This argument is valid only in bosonic string theory. In superstring theory, one has two kinds of two point functions: $\langle X^\mu X^\nu \rangle$ and $\langle \psi^\mu \psi^\nu \rangle$. Thus the metric appearing in the effective action is generically a combination of these. This combination can be different for every excited state, and actually that for the tachyon and the gauge field are different [42].

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JHEP09(2002)061

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